Lesson 15: Efficiently Solving Inequalities

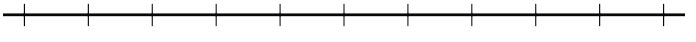
15.1: Lots of Negatives

Here is an inequality: $-x \geq -4$.

1. Predict what you think the solutions on the number line will look like.

2. Select **all** the values that are solutions to $-x \geq -4$:
   a. 3
   b. -3
   c. 4
   d. -4
   e. 4.001
   f. -4.001

3. Graph the solutions to the inequality on the number line:

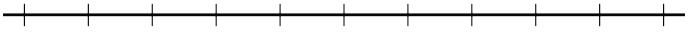


15.2: Inequalities with Tables

1. Let's investigate the inequality $x - 3 > -2$.

x	-4	-3	-2	-1	0	1	2	3	4
$x - 3$	-7		-5				-1		1

   a. Complete the table.
   b. For which values of x is it true that $x - 3 = -2$?
   c. For which values of x is it true that $x - 3 > -2$?
   d. Graph the solutions to $x - 3 > -2$ on the number line:



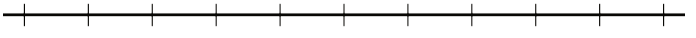
2. Here is an inequality: $2x < 6$.

a. Predict which values of x will make the inequality $2x < 6$ true.

b. Complete the table. Does it match your prediction?

x	-4	-3	-2	-1	0	1	2	3	4
$2x$									

c. Graph the solutions to $2x < 6$ on the number line:



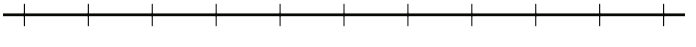
3. Here is an inequality: $-2x < 6$.

a. Predict which values of x will make the inequality $-2x < 6$ true.

b. Complete the table. Does it match your prediction?

x	-4	-3	-2	-1	0	1	2	3	4
$-2x$									

c. Graph the solutions to $-2x < 6$ on the number line:



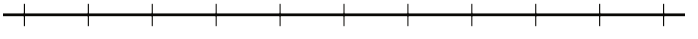
d. How are the solutions to $2x < 6$ different from the solutions to $-2x < 6$?

15.3: Which Side are the Solutions?

1. Let's investigate $-4x + 5 \geq 25$.
   a. Solve $-4x + 5 = 25$.

   b. Is $-4x + 5 \geq 25$ true when x is 0? What about when x is 7? What about when x is -7?

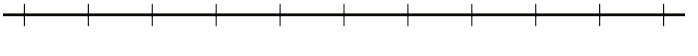
   c. Graph the solutions to $-4x + 5 \geq 25$ on the number line.



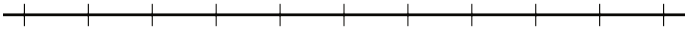
2. Let's investigate $\frac{4}{3}x + 3 < \frac{23}{3}$.
   a. Solve $\frac{4}{3}x + 3 = \frac{23}{3}$.

   b. Is $\frac{4}{3}x + 3 < \frac{23}{3}$ true when x is 0?

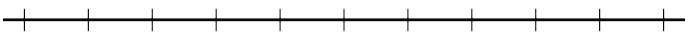
   c. Graph the solutions to $\frac{4}{3}x + 3 < \frac{23}{3}$ on the number line.



3. Solve the inequality $3(x + 4) > 17.4$ and graph the solutions on the number line.



4. Solve the inequality $-3\left(x-\frac{4}{3}\right) \leq 6$ and graph the solutions on the number line.



Are you ready for more?

Write at least three different inequalities whose solution is $x > -10$. Find one with x on the left side that uses a $<$.

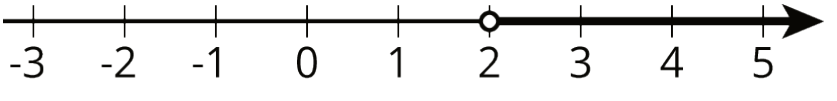
Lesson 15 Summary

Here is an inequality: $3(10-2x) < 18$. The solution to this inequality is all the values you could use in place of x to make the inequality true.

In order to solve this, we can first solve the related equation $3(10-2x) = 18$ to get the solution $x = 2$. That means 2 is the boundary between values of x that make the inequality true and values that make the inequality false.

To solve the inequality, we can check numbers greater than 2 and less than 2 and see which ones make the inequality true.

Let's check a number that is greater than 2: $x = 5$. Replacing x with 5 in the inequality, we get $3(10-2 \cdot 5) < 18$ or just $0 < 18$. This is true, so $x = 5$ is a solution. This means that all values greater than 2 make the inequality true. We can write the solutions as $x > 2$ and also represent the solutions on a number line:



Notice that 2 itself is not a solution because it's the value of x that makes $3(10-2x)$ *equal* to 18, and so it does not make $3(10-2x) < 18$ true.

For confirmation that we found the correct solution, we can also test a value that is less than 2. If we test $x = 0$, we get $3(10-2 \cdot 0) < 18$ or just $30 < 18$. This is false, so $x = 0$ and all values of x that are less than 2 are not solutions.